Handout on Ex Ante Evaluation in a Three Period Schooling Choice Model

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1 Model

We next describe a three period model that describes decisions made by parents about a child attending school. In the first two periods, parents make decisions about whether to send the child to school or to work. If a child works, he/she contributes to family income. If a child goes to school, he/she increases school attainment by one year. In the final time period, parents get utility from their child's school attainment.

Let

$$\begin{aligned} d(t) &= 1 \text{ if attend school in period t} \\ &= 0 \text{ if work and do not attend school} \end{aligned}$$

Also let $S(t)$ denote the child's schooling attainment in period t. Assume there are three time periods. In the first two time periods, parents decide whether their child attends school (i.e. decide on $d(1)$ and $d(2)$). In the third time period, the child is an adult and $d(3) = 0$.)

Parents utility is given by

$$U(3) = \alpha S(3),$$

where

$$\begin{aligned} S(3) &= d(1) + d(2) \\ &= S(2) + d(2). \end{aligned}$$

In periods 1 and 2, utility function is given by

$$U(t) = \alpha S(t) + (\beta_1 - \beta_2 k^s + \varepsilon^s(t))d_1(t) + C(t)$$

where k^s denotes the distance to school. Also, consumption $C(t)$ is

$$\begin{aligned} C(t) &= y^P + y^c(t)(1 - d(t)) \\ y^c(t) &= \gamma_0 - \gamma_1 k^c + \varepsilon^c(t), \end{aligned}$$

where y^P is the income of the parents and k^C denotes the distance to the city, which is a determinant of child wages.

The errors are assumed to be joint normally distributed:

$$\begin{pmatrix} \varepsilon^s(t) \\ \varepsilon^c(t) \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_s^2 & \sigma_{cs} \\ \sigma_{cs} & \sigma_c^2 \end{pmatrix}\right).$$

2 Identification of Model Parameters

We next analyze the decision rule in period 2, taken as given the period one choice $d(1)$. The value functions associated with the choices $d(2) = 1$ and $d(2) = 0$ are:

$$\begin{aligned} V^1(2) &= \alpha d(1) + \beta_1 - \beta_2 k^s + y^P + \varepsilon^s(2) + \delta\alpha(d(1)+1) \\ V^0(2) &= \alpha d(1) + y^P + \gamma_0 - \gamma_1 k^C + \varepsilon^c(2) + \delta\alpha d(1) \end{aligned}$$

and the parents choose

$$\begin{aligned} d(2) &= 1 \text{ iff } V^1(2) \geq V^0(2) \\ & \quad\text{or} \\ \varepsilon^c(2) - \varepsilon^s(2) &\leq (\beta - \gamma_0) - \beta_2 k^s + \gamma_1 k^c + \delta_\alpha \end{aligned}$$

Therefore,

$$\Pr(d(2) = 1) = \Phi\left(\frac{(\beta - \gamma_0) - \beta_2 k^s + \gamma_1 k^c + \delta\alpha}{\sigma_c^2 - 2\sigma_{cs} + \sigma_s^2}\right).$$

Assume that $y^c(2)$ is observed for all children, regardless of whether they work. Then $\gamma_0, \gamma_1, \sigma_c^2$ are identified from a regression of $y^c(2)$ on k^c. From the probit model, β_2, β_1, and $\alpha\delta$ are also identified. Note that having k^c (an exclusion restriction) in the child wage function is important to the identification argument.

If $y^c(2)$ not observed for children who do not work, then still identified both because of the exclusion restriction and because of normality.

Next, examine how the discount factor δ can be identified. Let Γ denote parameters that could be identified by the above argument. The value functions in period 1 are given by:

$$\begin{aligned} V^1(1) &= \beta_1 - \beta_2 k^s + y^P + \varepsilon^s(1) + \delta \text{ Emax } (V^1(2), V^2(2)|d(1) = 1) \\ V^0(1) &= y^P + \gamma_0 - \gamma_1 k^c + \varepsilon^c(1) + \delta \text{ Emax } (V^1(2), V^2(2)|d(1) = 0) \end{aligned}$$

$$\begin{aligned} \text{Emax}(V^1(2), V^2(2)|d(1) &= 1) = F_1(\alpha\delta; \Gamma) \\ \text{Emax}(V^1(2), V^2(2)|d(1) &= 0) = F_2(\alpha\delta; \Gamma) \end{aligned}$$

Parents choose school in the first period if

$$d(1) = 1 \text{ if } V^1(1) > V^0(1),$$

so

$$\Pr(d(1) = 1) = \Phi\left(\frac{(\beta - \gamma_0) - \beta_2 k^s + \gamma_1 k^c + \delta[F_1(\alpha\delta;\Gamma) - F_2(\alpha\delta;\Gamma)]}{\sigma_c^2 - 2\sigma_{cs} + \sigma_s^2}\right).$$

By functional form assumptions, δ can be separately identified from $\alpha\delta$.

3 Performing counterfactuals

Given the parameter estimates, we can perform counterfactuals. For example,

School Subsidy A school subsidy can be introduced into the constraint as

$$C(t) = y^P + y^c(t)(1 - d(t)) + bd(t)$$

and we can predict the school choices with and without the subsidy as above.

Graduation Bonus A graduation bonus given in period three can be introduced as

$$U(3) = \alpha S(3) + bI(S(3) = 2)$$

School Building A school building program could be seen as reducing the distance cost of traveling to school. For example, we could analyze decision-making with k^s set equal to 0 (e.g. a school in every village).