Problem Set #1

This problem asks you to calibrate a finite-horizon, discrete time, discrete outcome, dynamic stochastic fertility model (similar to that in Wolpin (1984)) and to carry out some simulation and estimation with the model. Assume that the decision-maker is a woman, who can decide to have a birth during each of twenty (two-year) decision periods. The utility function in any decision-making period is given by

$$U(C_t, N_t) = C_t - \frac{1}{2}\alpha_1 C_t^2 + (\alpha_2 + \varepsilon_t)N_t - \alpha_3 N_t^2 + \alpha_4 C_t N_t,$$

where C_t is consumption, N_t is number of children, and ε_t is a random shock to the utility from children. Assume that the utility value in the terminal period is 0.

The budget constraint is given by

$$C_t = y_t - p_n N_t,$$

where y_t is spouse's income. Assume that women have perfect foresight over their husband's income

$$y_t = \alpha_5 + \alpha_6 t.$$

Also, assume that $\varepsilon_t \tilde{} N(0, \sigma_\varepsilon^2)$.

(a) Pick parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and the variance of ε_t, under the assumption that $\alpha_4 < 0$ (consumption and children are substitutes), $\alpha_5 = 30,000$ and $\alpha_6 = 0$. Simulate the fertility decisions of 1000 women. Adjust your parameters so that people have on average 3-5 children over their lifetime.

(b) Keeping other parameters fixed at the values from (a), examine how fertility decisions change as you increase the price of children (p_n).

(c) Keeping other parameters fixed at the values in (a), examine how fertility decisions change as you increase the income profile, while keeping average income constant (e.g. decrease α_5 to 10,000 and increase α_6 to 2000).

(d) Keeping other parameters fixed at the values in (a), examine how the spacing of children changes as you increase the variance in the shock.

(e) For parts (a)-(d), plot the number of women who have a child at each age and the average number of children per family at each age. Also, tabulate the average duration (number of years) to first, second, and third birth.

(f) Now, treat the observations that you simulated under part (a) and pretend that they are your actual data. Write down the maximum likelihood estimation problem and estimate the parameters of your model using simulated maximum likelihood. How close do your estimates come to the values that you used for simulation?